



Single Satellite Doppler Localization with Law of Cosines (LOC)

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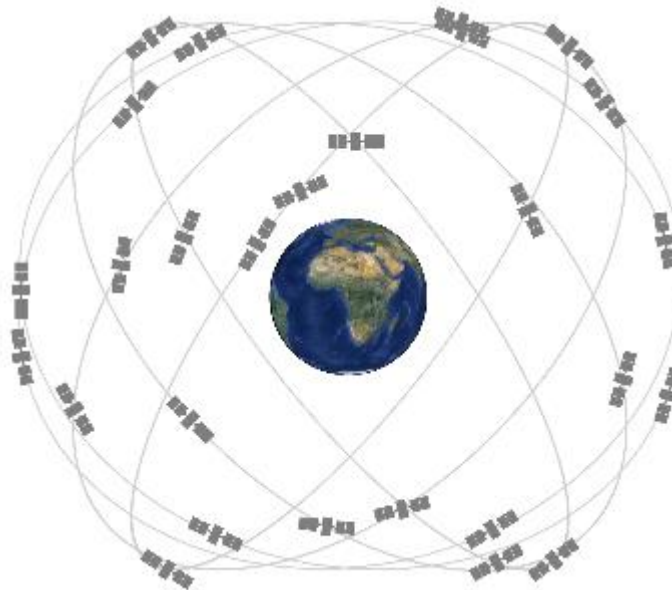
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Motivation and Problem Statement

Positioning on Other Planetary Bodies

- Traditional Earth Based Positioning Schemes rely on Ranging
- GPS constellation on Earth requires a minimum of 24 satellites in 6 orbit planes for 95% coverage¹
- Not currently attainable on other planetary bodies



Motivation and
Problem
Statement

Doppler
Localization using
Law of Cosines

Simulation
and Results

Conclusions

Applications

Motivation and Problem Statement

Positioning on Other Planetary Bodies

- Dead Reckoning
 - Visual Odometry
 - Bundle Adjustment
-
- All include aspects of Earth-in-loop post processing and take significant amounts of time²
-
- Current space proximity link radios do not provide ranging functions but can provide Doppler measurements

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Elements of the Problem Statement

- Achieving localization of a surface user with:
 - Only Doppler Measurements
 - Using existing space proximity link radios
 - Static Reference Station on Planetary Surface
 - With known location
 - Can receive from satellite and transmit to user
 - Minimum of One Satellite
 - With known ephemeris
- Reduce overall navigational cost by enabling relative positioning with as little as one satellite
- Relax requirements on user and satellite hardware

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Doppler as a Datatype

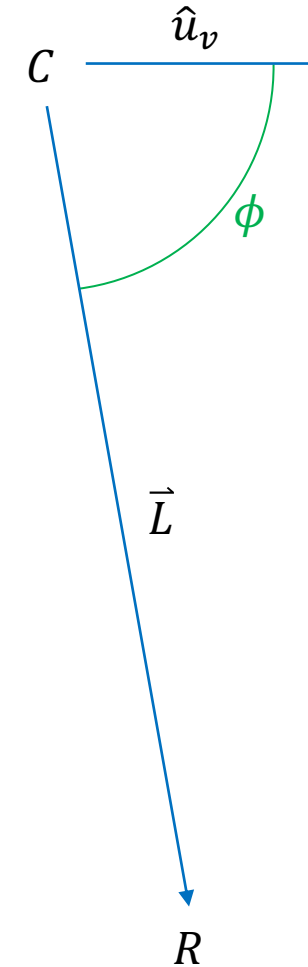
- Doppler shift \rightarrow Range Rate \rightarrow Angle θ

$$\cos\phi = -\frac{\text{RangeRate}_R}{\|\vec{V}_{sat}\|}$$

- Definition of Angle between Vectors

$$\cos\phi = \frac{\vec{L} \cdot \hat{u}_v}{\|\vec{L}'\|} \rightarrow \boxed{\|\vec{L}'\| = \frac{\vec{L} \cdot \hat{u}_v}{\cos\phi}}$$

- Pseudo – Pseudorange Expression



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Law of Cosines (LOC)

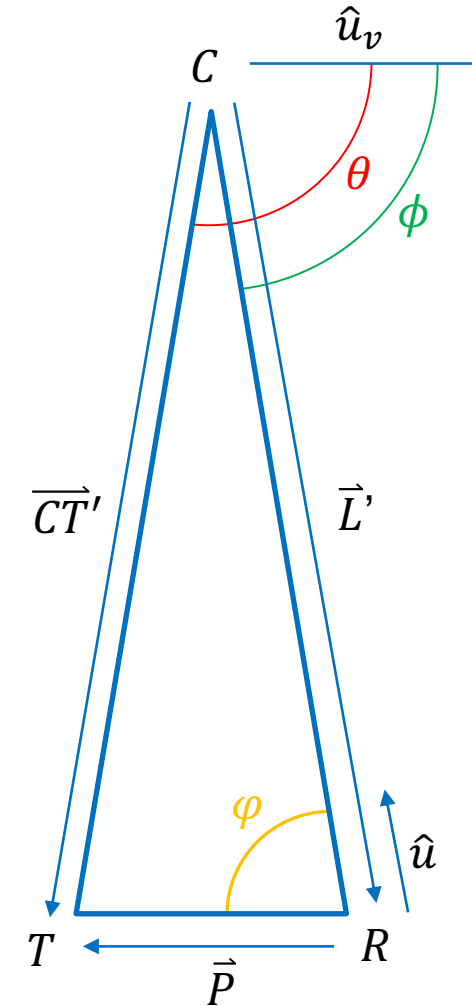
- Pseudo – Pseudorange Expressions for User and Reference Station

$$\|\vec{L}'\| = \frac{\vec{L} \cdot \hat{u}_v}{\cos\phi}, \quad \|\vec{CT}'\| = \frac{(\vec{L} + \vec{P}) \cdot \hat{u}_v}{\cos\theta}$$

$$\cos\phi = \frac{\vec{P} \cdot \hat{u}}{\|\vec{P}\|}$$

- Law of Cosines

$$\|\vec{CT}'\|^2 = \|\vec{L}'\|^2 + \|\vec{P}\|^2 - 2\|\vec{L}'\|\|\vec{P}\|\cos\phi$$



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Minimizing a Cost Function

$$f(\vec{P}) = \|\vec{L}'\|^2 + \|\vec{P}\|^2 - 2\|\vec{L}'\|\|\vec{P}\|\cos\varphi - \|\vec{CT}'\|^2$$

$$\|\vec{L}'\| = \frac{\vec{L} \cdot \hat{u}_v}{\cos\phi}, \quad \|\vec{CT}'\| = \frac{(\vec{L}' + \vec{P}) \cdot \hat{u}_v}{\cos\theta}$$

$$\cos\varphi = \frac{\vec{P} \cdot \hat{u}}{\|\vec{P}\|}$$

$$f(\vec{P}) = \left(\frac{\vec{L} \cdot \hat{u}_v}{\cos\phi} \right)^2 + \|\vec{P}\|^2 - 2 \left(\frac{\vec{L} \cdot \hat{u}_v}{\cos\phi} \right) (\vec{P} \cdot \hat{u}) - \left(\frac{(\vec{L} + \vec{P}) \cdot \hat{u}_v}{\cos\theta} \right)^2$$

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Addition of the Surface Constraint

- \vec{P} vector has 3 scalar unknowns
- Each Doppler measurement instance adds one scalar known
- Reduce one of the unknowns with the Surface Constraint
 - Knowledge of the user's altitude and with an accurate topocentric map of the user's location
 - This can be used as a range measurement from a faux satellite located at the center of the planet
- Multiple measurements can be made from one satellite over time
 - Two measurements taken 30 minutes apart with a static user
 - Single Satellite Multiple Measurements (SSMM)

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Scenario and Methodology

- Lunar Relay Satellite (LRS) Orbit
- Reference Station on the South Pole
- User 10 km away

Table 1: Modelled Gaussian Random Error

Error	Sigma (σ)
Satellite Ephemeris (3D)	5 m
Satellite Velocity Vector (3D)	1 cm/s
Doppler Measurement	0.005 Hz

Table 2: Convergence Properties

Convergence	0.01 cm
Convergence Iteration Limit	25
Monte Carlo Simulations	10,000

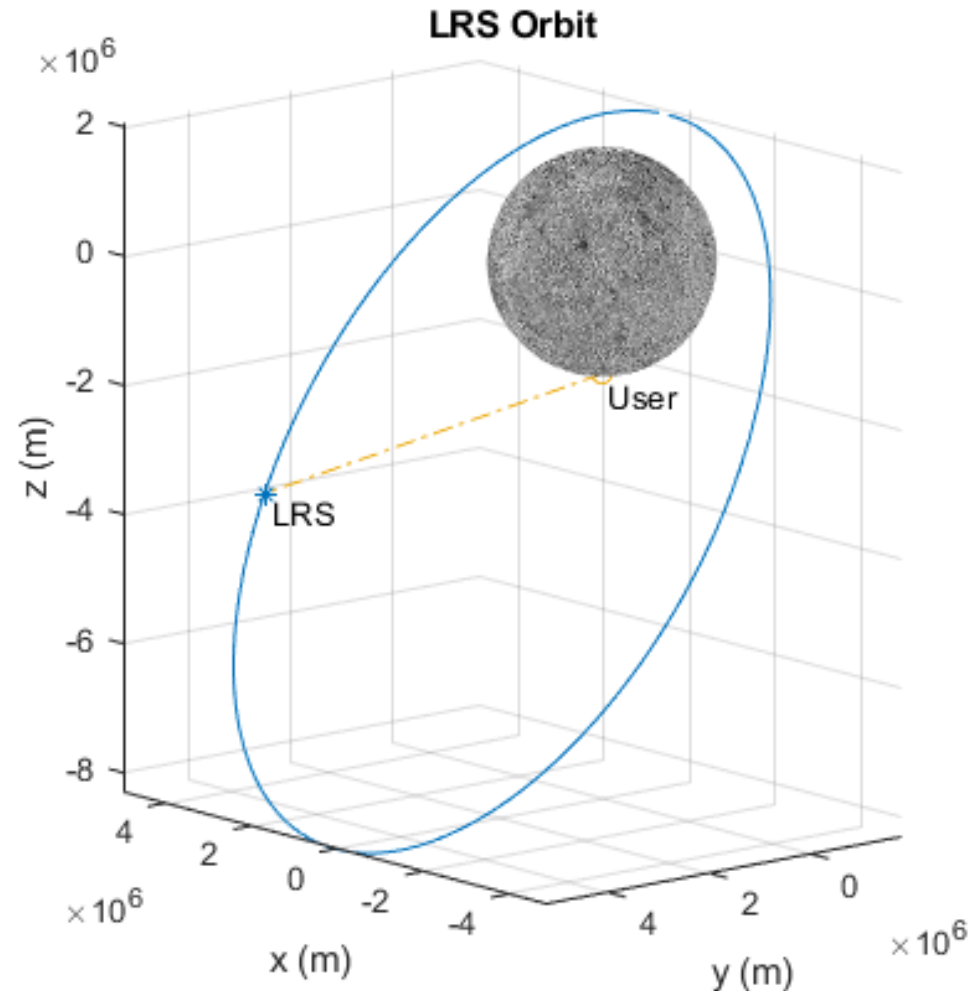


Figure 1: Lunar Relay Satellite (LRS)
Frozen 12 Hour Orbit

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Results Over the Entire Pass

- Increase in 3D Positioning Error at apoapsis
 - Lowest Velocity and therefore smaller Doppler shift measurement
- Beginning of the pass had the greatest velocity
 - 3D position Root Mean Square Error (RMSE) of approximately 22 meters

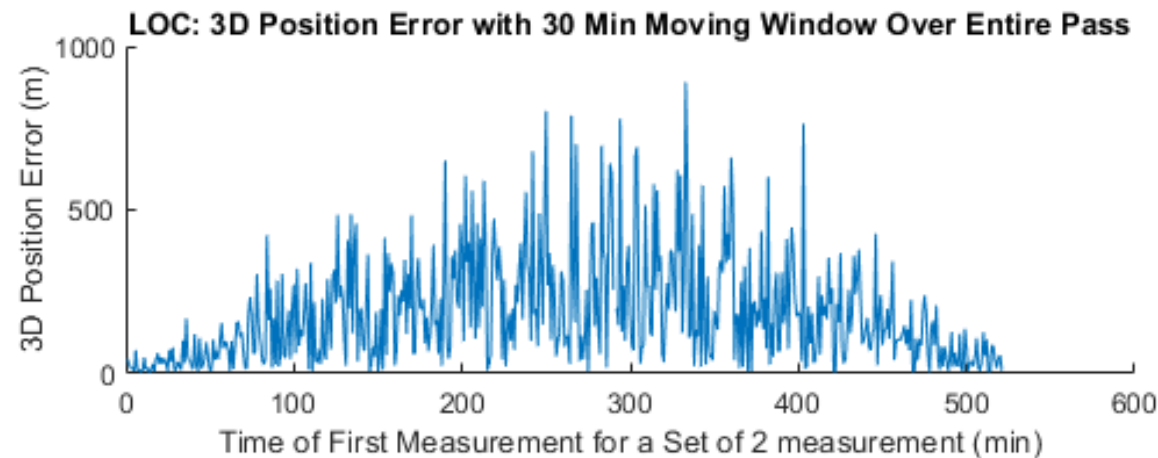


Figure 2: 3D Positioning Error with 2 Measurements over 30 Minutes

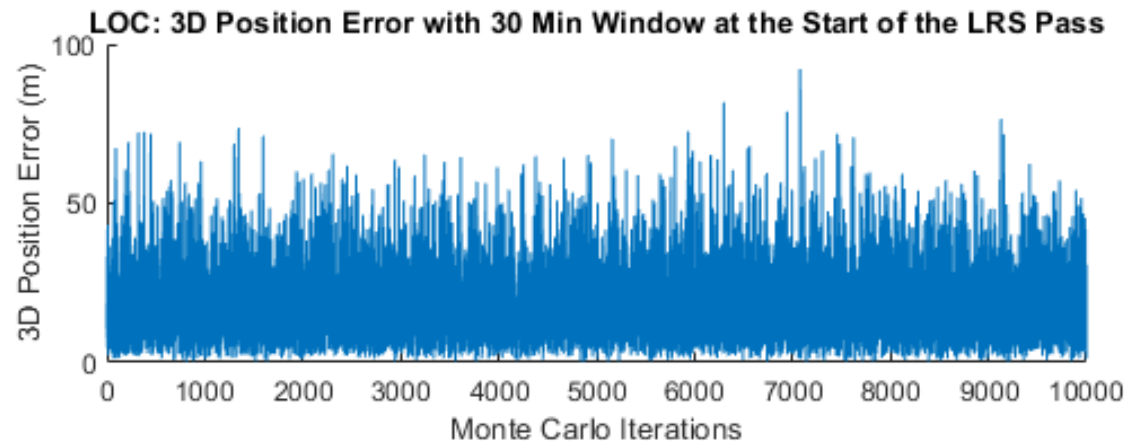


Figure 3: 3D Positioning Error over 10,000 Monte Carlo Simulations

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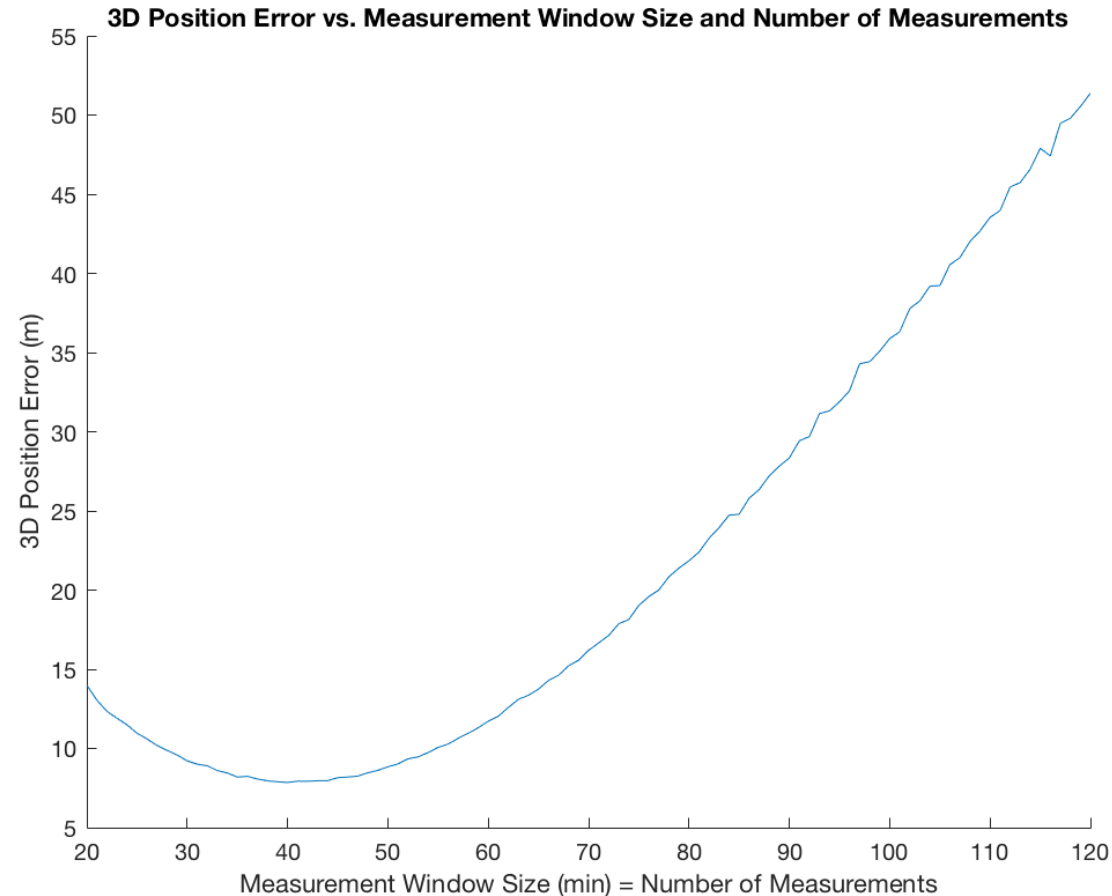
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Minimizing Error

- Increase number of measurements and length of measurement window
- Local Minimum
 - 38 measurements
 - 40 minute window
- 3D RMSE of ~7 meters



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**Simulation
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Conclusions

- The LOC technique can be used to achieve localization with as few as one satellite, a reference station, and only Doppler measurements
 - 3D Position RMSE as low as 7 meters
- Reduces navigational cost and relaxes hardware requirements on user and orbiter
 - Can be performed using existing hardware
- Allows for near real time localization and navigation along a planetary surface without Earth-in-loop post processing

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- Expanded to any planetary body with an orbiter and a reference station
 - MRO with a Habitat Base on Mars
- Can be used for emergency situations with any satellite with a radio and a reference station
- Joint Doppler and Ranging (JDR) techniques³
 - Enable real-time localization with increased precision, and is more robust with respect to the orbiter - user geometry

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Jet Propulsion Laboratory
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Appendix

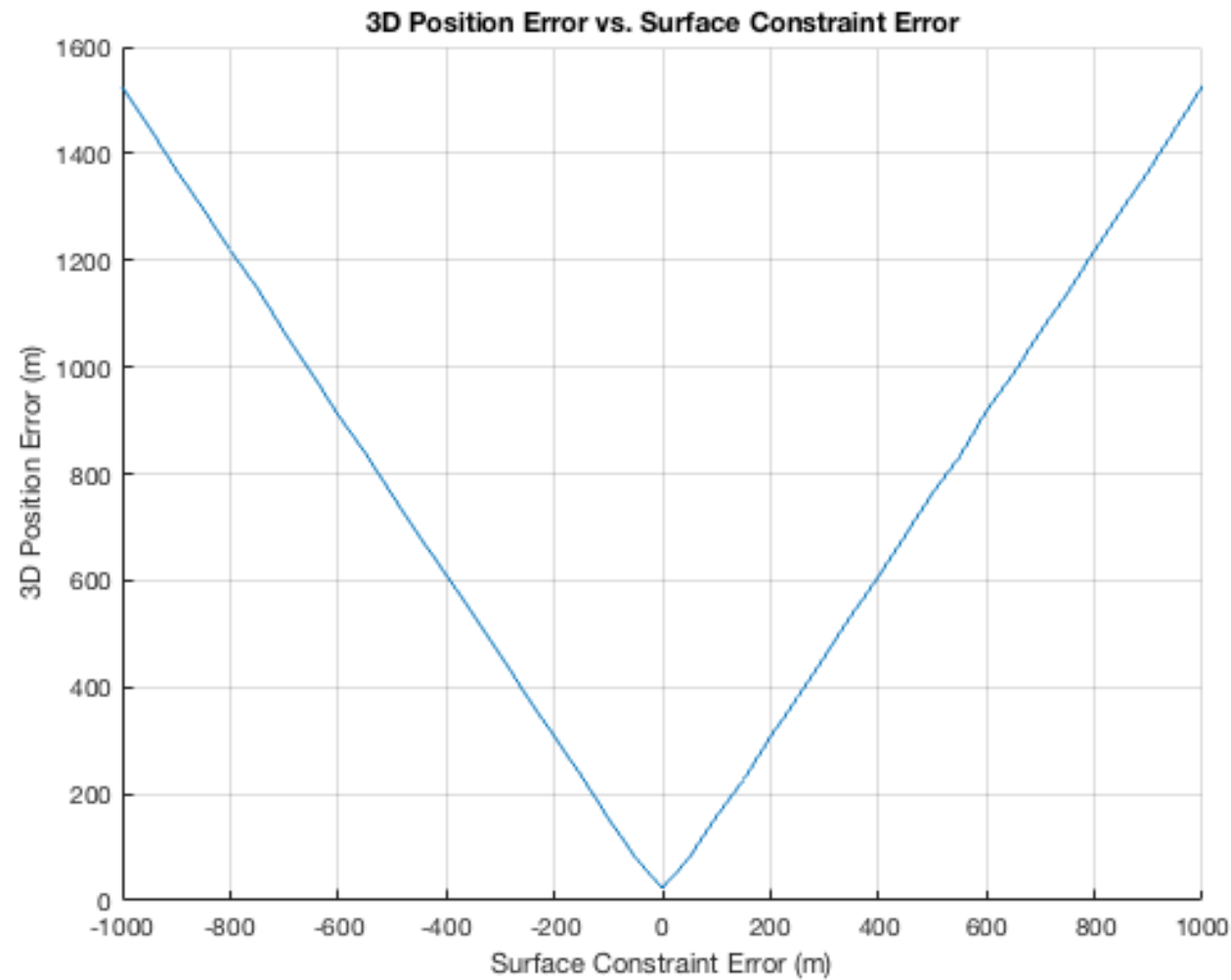


Figure A.1: 3D Position Error vs. Surface Constraint Error

Appendix

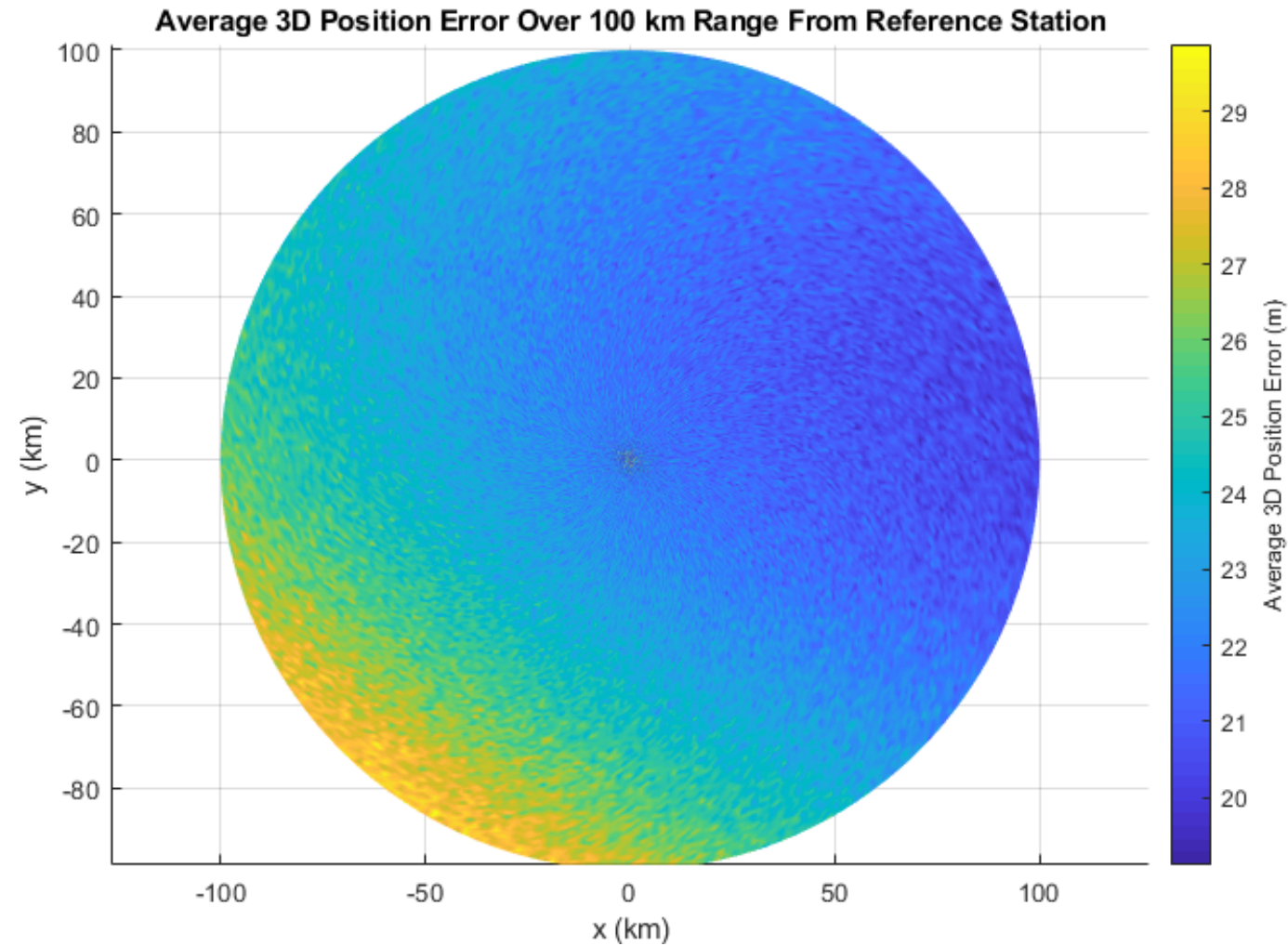


Figure A.2: 3D Position Error vs. Relative Location of User with the Reference Station at the center of the circle (0,0)

Appendix

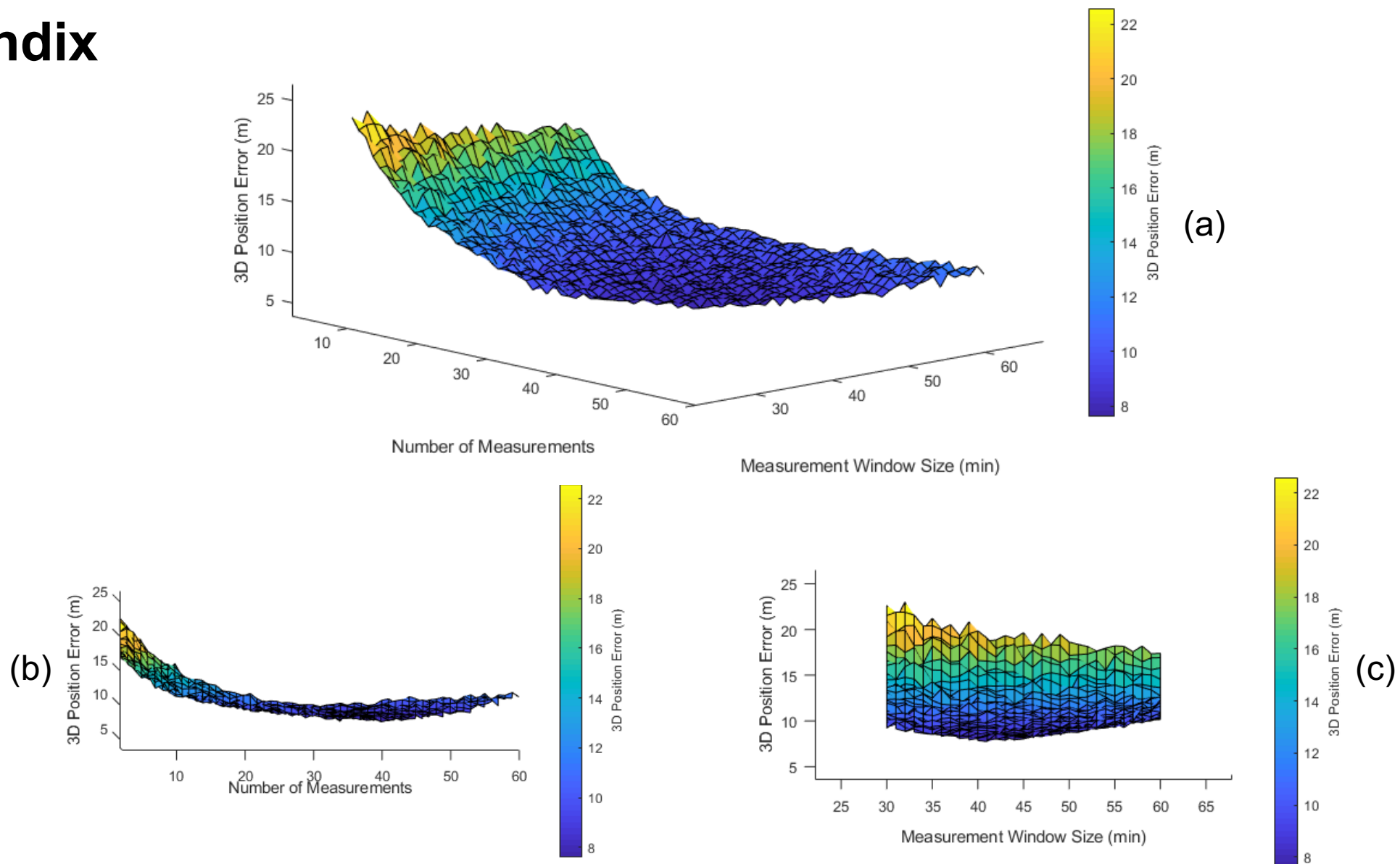


Figure A.3: 3D Position Error vs. Measurement Window Size vs. Number of Measurements per Window at the beginning of the LRS pass. (a) 3D view, (b) side view of relationship with Number of Measurements, (c) front view of relationship with Measurement Window Size